The Physics of Time Travel

In memory of my father, who lives

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Abstract

The principle of General Covariance allows to understand the dynamics of accelerated reference frames using the formalism of General Relativity. Using this principle, the metric, the affine connection and the equations of the dynamics in an accelerated reference frame, in the absence of gravity, are deduced easily. The equations obtained explain: the differences between gravity and inertia, what is time and how it is possible to travel in time. Finally, the relativistic equations of motion of the rotating perfect fluid without pressure are solved, calculating the external force applied to go back in time. In the Appendix the formalism is generalized to the case in which gravity is present.

INTRODUCTION

There are several ways in which time travel arises in General Relativity although in brief, there are basically two families of solutions: rotating space-times and space-times which allow for faster than light travel.

Among rotating space-time induced time machines, the first geometry to be discovered to have CTC involves a cosmological solution generated by rotating dust [1]. The Kerr space-time and the Tomimatsu-Sato space-time [2] all contain CTCs in their inner regions. CTCs can be generated by infinitely long rotating cylinders of matter, known as Tipler cylinders [3]. Alternatively, two cosmic strings passing one-another generate CTC [4], [5].

Beyond accidentally generating CTCs as a consequence of angular momentum, there are families of geometries which have been deliberately designed to contain CTCs. The Ori [6] and Ori-Soen [7] space-times both probe the physicality of generating a region with CTCs. If the two mouths of a traversable wormhole are properly accelerated with respect to one another, the twin paradox can result in a CTC path between the mouths [8-11].

Finally, space-times where timelike curves can travel along superluminal (as described by a distant observer) trajectories can be used to generate closed timelike curves. The Alcubierre warp drive [12] can be used to generate CTCs [9], [13]; as does the Krasnikov tube [10], [14].

If there are so many ways to go about it, why have we never seen evidence of retrograde time travel in our universe? The answer depends on the particular CTC containing geometry. Space-times permitting superluminal travel often require a violation of the classical energy conditions [15], [16], and are therefore not classically realizable. The CTC region in the Kerr space-time lay behind an event horizon, the Tomimatsu-Sato geometries are not recognized as the end state of gravitational collapse, and the other mentioned models require infinitely large distributions of matter.

However, there is an alternative based on the Principle of Equivalence of Gravitation and Inertia postulated by Einstein in the General Relativity. As we know, it is easier to handle inertia appearing with the accelerated motion than gravity, although the effects in both cases must be locally equivalent. Due to this, we can find metrics originated as a consequence of accelerated motion which allow to travel to the past.

In general, the invariance under Lorentz transformations is satisfactory when the mass does not appear in the equations of the field, as in electromagnetism. If mass is present, as in the case of mechanics or gravity then general invariance is more appropriate.
The General Covariance is highly adequate to find the equations of dynamics in accelerated reference frames, the metric and the affine connection in a space-time without gravity. Its main advantage is that the coordinate transformations between observers in motion appear explicitly in the equations of dynamics. This allows to obtain the dynamics of the motion from known coordinate transformations constrained by the available degrees of freedom.
This paper shows how the travel to the past is possible using a simple model of rotating perfect fluid without pressure. Firstly, it is assumed that the Gravitational field created by the fluid is negligible, which implies flat space-time. The novelty is that in this flat space-time CTCs are not created, which allows to travel at any time in the past, nor huge amounts of matter that curve space-time are needed, nor it resorts to exotic matter or negative energy. Instead of that, the relativistic equations of motion of the fluid are solved analytically, which allows to discuss the results. Finally we consider the case when the Gravitational field of the fluid is important and therefore gravity is present.
Sections I to IV consider the motion in space-time without gravity. Section I studies the moving reference frames and the field. Section II, analyses the rotating reference frames. In section III, the previous results are applied to study the motion of the rotating perfect fluid without pressure. In section IV, the force applied on this model of rotating fluid is calculated to go back in time. Finally, the Appendix connects with the equations of the field and gravity.

I. MOVING REFERENCE FRAMES

Unfortunately, there is no general agreement on how to deal with accelerated motion in a relativistic manner. For instance, the treatment advocated in [17-19] is incompatible with most of the methods of relativistic reference frames [20-27] and it is also inadequate from the practical standpoint involving the transformation between the experimental (accelerated) and laboratory (inertial) frames. In general, it is not clear how to write a suitable explicit form of the metric tensor of the accelerated reference frame and construct coordinate transformations between the inertial and accelerated frames, especially when high accuracy is required.
For these reasons, in this paper, the coordinate transformations used to construct the metric are presented in a general form that relies on a set of functions which validity will be determined in the subsequent sections. Moreover, we will use a different gauge to constrain the available degrees of freedom.

Equations of Motion

When relativistic dynamics is studied, it is convenient to define two observers linked to two different reference frames: one that is considered at rest and the other moving with respect to the first. From now on they are going to be known as reference frame at rest S´ and moving reference frame S.
Let S´ be an inertial reference frame at rest in which the laws of Special Relativity are valid globally. In this frame the metric $\eta_{\mu\nu}$ is the Minkowski metric. An observer located in S´ defines Cartesian coordinates $x'^{\mu} = (t', x', y', z')$ to measure events. Let S

be a moving reference frame with respect to S´ in which other coordinates $x^{\mu} = (x^0, x^1, x^2, x^3)$ are used to measure the events.
The equations of motion in S´ of a fluid moving under the action of an external force are according to the laws of Special Relativity

$$f'^{\mu} = \frac{\partial T'^{\mu\alpha}}{\partial x'^{\alpha}} \tag{1.1}$$

The equations of motion must be invariant under any coordinate transformations.
Suppose that the coordinate transformations between S´ and S are continuous functions of the form $x'^{\mu} = x'^{\mu}(x^0, x^1, x^2, x^3)$.These transformations must be global, covering all space-time or at least the volume of the fluid.
The inverse transformations are functions of the form $x^{\mu} = x^{\mu}(t', x', y', z')$. The necessary and sufficient condition for this to be true is that the Jacobian of the transformation must be nonzero.[1]
Then, carrying out any coordinate transformations from S´ to S, in the equations of motion (1.1) , we can find the equations of motion of a fluid in any reference frame, for example S

$$f^{\mu} = T^{\mu\alpha}{}_{;\alpha} = \frac{\partial T^{\mu\alpha}}{\partial x^{\alpha}} + \Gamma^{\alpha}_{\alpha\beta} T^{\mu\beta} + \Gamma^{\mu}_{\alpha\beta} T^{\alpha\beta} \tag{1.2}$$

where

$$f^{\mu} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} f'^{\alpha}. \tag{1.3}$$

The metric is

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{\partial x'^{\beta}}{\partial x^{\nu}} \tag{1.4}$$

and the affine connection is

$$\Gamma^{\lambda}_{\mu\nu} = \frac{\partial x^{\lambda}}{\partial x'^{\alpha}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \tag{1.5}$$

or

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left(\frac{\partial g_{\mu\alpha}}{\partial x^{\nu}} + \frac{\partial g_{\nu\alpha}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\alpha}} \right). \tag{1.6}$$

The principle of General Covariance tells us that Eqs. (1.2) are true in any Inertial or Gravitational field. It is important to note that they explicitly show the coordinate transformations between S´ and S through the affine connection (1.5).
In order to solve Eqs. (1.2) we need to fix the reference frame, that is, the gauge. If the reference frame S is chosen so that the fluid remains at rest with respect to it, then

$$\frac{dx^i}{d\tau} = 0. \tag{1.7}$$

Equations (1.7) do not fix the Gauge alone, that is, the reference frame. It is necessary to add another condition: the time-time component of the metric has to be invariant[2]

$$g_{tt} = \eta_{tt} = -1. \tag{1.8}$$

Meaning of Time

[1] Except maybe for some particular choices of coordinates which are called “singular”.
[2] This condition is needed for both the fluid and the reference frame S comove in time.

As it is well known, the three spatial dimensions are characterized by freedom of motion. This is not so in the case of time as it is demonstrated below.
Equation (1.8) and the time-time component of Eq. (1.4) give

$$\left(\frac{\partial x'}{\partial t}\right)^2+\left(\frac{\partial y'}{\partial t}\right)^2+\left(\frac{\partial z'}{\partial t}\right)^2=-1+\left(\frac{\partial t'}{\partial t}\right)^2. \tag{1.9}$$

On the other hand, the components of the velocity of S with respect to S´ are

$$v'^i=\frac{\dfrac{\partial x'^i}{\partial t}}{\dfrac{\partial t'}{\partial t}} \tag{1.10}$$

since S is at rest in its own reference frame.
In addition, the magnitude of the velocity can be calculated with Eqs. (1.10) and (1.9)

$$v'=\sqrt{v'^i v'^i}=\sqrt{\frac{-1+\left(\dfrac{\partial t'}{\partial t}\right)^2}{\left(\dfrac{\partial t'}{\partial t}\right)^2}}\le 1. \tag{1.11}$$

This equation shows, as expected, that it can never be greater than unity, that is, any reference frame can't move at a speed greater than the speed of light, with respect to S´.
From Eq. (1.11), it is found

$$\frac{\partial t'}{\partial t}=\frac{1}{\sqrt{1-v'^2}}=\gamma'. \tag{1.12}$$

This equation explains time dilation that occurs when velocity increases and it is a direct consequence of the choice made in Eq. (1.8). It shows that, in the absence of gravity, a clock at rest relative to another will run at the same rate, while one that moves relative to first will indicate its own time, different from the other. It shows that motion in time depends on motion in space.[3] In particular, the velocity of motion in time $\frac{\partial t'}{\partial t}$ of S with respect to S´ depends on its spatial velocity v'. Consequently, time is a dimension in which there is no freedom of motion because motion in time is not independent of motion in space, but depends on it as in Eq. (1.12).This dependence makes it possible to determine the spatial velocity of a fluid to go back in time. The answer will be found in the following sections.

Inertial Field

The curvature tensor in S can be defined as a function of the affine connection

$$R^{\lambda}{}_{\mu\nu\sigma}=\frac{\partial\Gamma^{\lambda}_{\mu\nu}}{\partial x^{\sigma}}-\frac{\partial\Gamma^{\lambda}_{\mu\sigma}}{\partial x^{\nu}}+\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\sigma\alpha}-\Gamma^{\alpha}_{\mu\sigma}\Gamma^{\lambda}_{\nu\alpha}. \tag{1.13}$$

Since the curvature tensor in S´ is clearly zero, it follows from the laws of transformation of the tensors that it is also zero in any other reference frame

$$R^{\lambda}{}_{\mu\nu\sigma}=0. \tag{1.14}$$

This result implies that space-time in S is also flat instead of curved and that gravity does not appear either in the moving reference frame S.

[3] As we know time also depends on the position in a gravitational field.

The Ricci tensor is obtained by contraction of the curvature tensor and it is also zero according to Equation (1.14)

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu} = 0. \tag{1.15}$$

This equation shows that the field in S has no sources but it is originated only by the motion of S with respect to S´, that is, the inertial field has not sources which curve space-time.

II. ROTATING REFERENCE FRAMES

There is no agreement either on how to establish the relativistic transformation to rotating frames. The paper of Klauber reviews different theories of relativistic rotation [28].
For this reason, the coordinate transformations used here to construct the metric in rotating frames are chosen using the only available degree of freedom and their validity will be determined later.

Coordinate Transformations

A rotating reference frame S is an accelerated reference frame with an interesting property: it moves in the same place. In this case it is convenient to define cylindrical coordinates $x^{\mu} = (t, r, \theta, z)$ in S for practical reasons.
The coordinate transformations from the reference frame at rest S´ to the rotating reference frame S are

$$\left.\begin{aligned} t' &= t'(t,r) \\ x' &= r cos\left[\theta - \varphi(t)\right] \\ y' &= r sen\left[\theta - \varphi(t)\right] \\ z' &= z \end{aligned}\right\}. \tag{2.1}$$

The validity of these transformations can be verified as solutions of the equations of motion obtained in Section III.

The Metric Tensor

The metric in S can be calculated with Eqs. (1.4) and (2.1)

$$g_{tt} = -\left(\frac{\partial t'}{\partial t}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -1, g_{tr} = -\frac{\partial t'}{\partial t}\frac{\partial t'}{\partial r}, g_{t\theta} = -r^2\frac{d\varphi}{dt}, g_{tz} = 0,$$

$$g_{rr} = 1 - \left(\frac{\partial t'}{\partial r}\right)^2, g_{r\theta} = 0, g_{rz} = 0, \tag{2.2}$$

$$g_{\theta\theta} = r^2, g_{\theta z} = 0, g_{zz} = 1$$

where Eq. (1.8) has been used in the first.
The first of Eqs. (2.2) can be written as

$$\frac{\partial t'}{\partial t} = \pm\sqrt{1 + r^2\left(\frac{d\varphi}{dt}\right)^2}. \tag{2.3}$$

This equation has two solutions.[4] The positive solution implies that t' increases with t, which is known as time dilation. In section IV we will see when negative solution is

[4] From now on, both signs will be shown except in the case when the choice of one of them is justified.

admissible, which ensures a decrease of t' as t increases, what it would allow to go back in time.

The Affine Connection

The affine connection can be calculated through Eqs. (1.6), with the metric (2.2) and its derivatives and the inverse of the metric tensor

$$\Gamma^r_{tt} = -r\left(\frac{d\varphi}{dt}\right)^2, \Gamma^t_{rt} = \frac{r\left(\frac{d\varphi}{dt}\right)^2}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}, \Gamma^t_{\theta t} = -\frac{r\frac{\partial t'}{\partial t}\frac{\partial t'}{\partial r}\frac{d\varphi}{dt}}{1+r^2\left(\frac{d\varphi}{dt}\right)^2},$$

$$\Gamma^t_{rr} = \frac{\frac{\partial t'}{\partial t}\frac{\partial^2 t'}{\partial r^2}}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}, \Gamma^t_{\theta\theta} = \frac{r\frac{\partial t'}{\partial t}\frac{\partial t'}{\partial r}}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}, \Gamma^r_{t\theta} = r\frac{d\varphi}{dt},$$

$$\Gamma^\theta_{tr} = -\frac{\frac{d\varphi}{dt}}{r\left[1+r^2\left(\frac{d\varphi}{dt}\right)^2\right]}, \Gamma^\theta_{t\theta} = -\frac{r\frac{\partial t'}{\partial t}\frac{\partial t'}{\partial r}\left(\frac{d\varphi}{dt}\right)^2}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}, \Gamma^r_{\theta\theta} = -r,$$

$$\Gamma^\theta_{rr} = \frac{\frac{\partial t'}{\partial t}\frac{\partial^2 t'}{\partial r^2}\frac{d\varphi}{dt}}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}, \Gamma^\theta_{r\theta} = \frac{1}{r}, \Gamma^\theta_{\theta\theta} = \frac{r\frac{\partial t'}{\partial t}\frac{\partial t'}{\partial r}\frac{d\varphi}{dt}}{1+r^2\left(\frac{d\varphi}{dt}\right)^2}. \qquad (2.4)$$

The rest of the components are zero.

III. ROTATING PERFECT FLUID WITHOUT PRESSURE

Equations of Motion in the Rotating Reference Frame

In this section we apply the above results to the simplest model of fluid, that is, a perfect fluid without pressure. In this case the energy-momentum tensor of the perfect fluid is

$$T^{\mu\nu} = \rho\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}. \qquad (3.1)$$

If the fluid is at rest in the rotating reference frame S, Eqs. (1.7), (1.8) and (3.1) give

$$T^{tt} = \rho; \quad T^{ti} = T^{it} = 0; \quad T^{ij} = 0. \qquad (3.2)$$

In addition, if the mass-energy is conserved in S´ the continuity equation of the fluid must be[5]

[5] This equation is not covariant so it is only valid in the reference frame at rest S´.

$$f'^{t} = 0 \tag{3.3}$$

and since the components of the density of the external force f'^{μ} can be calculated through Eqs. (1.3)

$$f'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} f^{\alpha} \tag{3.4}$$

the Eq. (3.3) give

$$\rho = \rho_i \frac{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2_{t=t_i}}{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2}. \tag{3.5}$$

This equation shows that the proper energy density of the fluid ρ is always positive or zero.
The components of the density of the external force applied to the fluid in S are calculated using Eqs. (1.2), (2.4), (3.2) and (3.5)

$$f^{t} = \pm \rho_i r \frac{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2_{t=t_i}\right] \sqrt{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2} \frac{\partial t'}{\partial r} \left(\frac{d\varphi}{dt}\right)^2}{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2\right]^2}, \tag{3.6}$$

$$f^{r} = -\rho_i r \frac{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2_{t=t_i}\right] \left(\frac{d\varphi}{dt}\right)^2}{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2}, \tag{3.7}$$

$$f^{\theta} = \rho_i \frac{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2_{t=t_i}\right] \left[\pm r \sqrt{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2} \frac{\partial t'}{\partial r} \left(\frac{d\varphi}{dt}\right)^3 - \frac{d^2\varphi}{dt^2}\right]}{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2\right]^2}, \tag{3.8}$$

$$f^{z} = 0. \tag{3.9}$$

The rotational motion is only due to the radial f^{r} and the tangential f^{θ} components of the density of the external force given by Eqs. (3.7) and (3.8) and from now on, only these two components will be referred to.

Equations of Motion in the Reference Frame at Rest

It is also convenient to determine the motion of the rotating fluid in the reference frame S′. The inverses of Eqs. (2.1) allow to obtain the density of the external force directly in S′ using Eqs. (3.3), (3.4), and (3.6) to (3.9)

$$f'^{t} = 0, \tag{3.10}$$

$$f'^{r} = f^{r}, \tag{3.11}$$

$$f'^{\theta} = -\rho_i \frac{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2_{t=t_i}\right] \frac{d^2\varphi}{dt^2}}{\left[1 + r^2 \left(\frac{d\varphi}{dt}\right)^2\right]^2}, \tag{3.12}$$

$$f'^{z} = f^{z} = 0. \tag{3.13}$$

Also for the same reasons given for S, from now on only the radial f'^{r} and tangential f'^{θ} components of the density of the external force will be referred to.
In addition, introducing Eq. (2.3) into Eq. (1.11) gives the velocity of S

$$v' = \frac{r \frac{d\varphi}{dt}}{\sqrt{1 + r^2 \left(\frac{d\varphi}{dt}\right)^2}} \leq 1. \tag{3.14}$$

If the angular velocity is constant $\frac{d\varphi}{dt} = \omega$ and the fluid is initially at rest at $t_i = t_0$ in S in which the density is ρ_0, then the relativistic centripetal force given by Eqs. (3.11) and (3.7) is different from the classical law

$$f'^{\,r}_{rel} = -\rho_0 \frac{\omega^2 r}{1 + \omega^2 r^2} \tag{3.15}$$

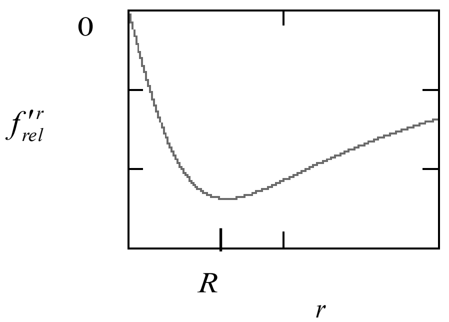

FIGURE 1. f'^{r}_{rel} as function of r at constant angular velocity ω.

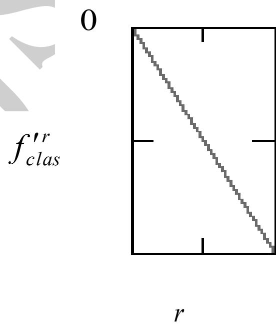

FIGURE 2. Classical law.

IV. TIME TRAVEL

In this section we apply the results obtained in the previous sections to determine the dynamics of the rotating perfect fluid without pressure to go back in time. This means finding the external force applied on the fluid to go back in time with rotational motion. For this purpose the only degree of freedom of a rotating perfect fluid, that is, the angular velocity $\frac{d\varphi}{dt}$ is fixed. This variable is chosen so that the associated time shift obtained from Eq. (2.3) allows the fluid to go back in time. The travel will take five legs. The procedure is the following: first, we fix the angular velocity $\frac{d\varphi}{dt}$ and second, we calculate $\frac{\partial t'}{\partial t}$ through Eq. (2.3), integrating the last we obtain $t' = t'(t, r)$ and deriving this we calculate $\frac{\partial t'}{\partial r}$. In addition, we use Eq. (3.5). Then, we insert these results into the equations of motion (3.7) and (3.8). Occasionally, we will use Eqs. (3.11) and (3.12) to show the motion in S´.

Temporal Acceleration

At the beginning, the observers located in S´ and S are in an inertial reference frame in which the Minkowski metric is valid. This allows synchronizing the clocks of both frames according to the criterion of the light signals proposed by Einstein [29].
The first leg of time travel occurs during the interval $t_0 \le t \le t_1$ for an observer located in S. If the reference frame S is initially at rest at $t = t_0$ the simplest function $\frac{d\varphi}{dt}$ is

$$\frac{d\varphi_I}{dt} = a_1(t - t_0) \tag{4.1}$$

where a_1 is a constant that can be chosen so that

$$a_1 > 0. \tag{4.2}$$

In addition

$$\frac{\partial t'_I}{\partial t} = \sqrt{1 + a_1^2 r^2 (t - t_0)^2}\,, \tag{4.3}$$

$$t'_I = t'_0 + \frac{t - t_0}{2}\sqrt{1 + a_1^2 r^2 (t - t_0)^2} + \frac{1}{2a_1 r} Ln\left[a_1 r(t - t_0) + \sqrt{1 + a_1^2 r^2 (t - t_0)^2}\right] \tag{4.4}$$

and

$$\frac{\partial t'_I}{\partial r} = \frac{t - t_0}{2r}\sqrt{1 + a_1^2 r^2 (t - t_0)^2} - \frac{1}{2a_1 r^2} Ln\left[a_1 r(t - t_0) + \sqrt{1 + a_1^2 r^2 (t - t_0)^2}\right] \ge 0. \tag{4.5}$$

We obtain for this fluid in the reference frame S during the first leg of the time travel

$$f_I^r = -\frac{\rho_0 a_1^2 r (t - t_0)^2}{1 + a_1^2 r^2 (t - t_0)^2}\,, \tag{4.6}$$

$$f_I^\theta = \rho_0 \frac{r\sqrt{1 + a_1^2 r^2 (t - t_0)^2}\,\frac{\partial t'_I}{\partial r}\,a_1^3 (t - t_0)^3 - a_1}{\left[1 + a_1^2 r^2 (t - t_0)^2\right]^2} \tag{4.7}$$

where $\frac{\partial t'_I}{\partial r}$ is given by Eq. (4.5).

Time Reversal

The second leg of time travel occurs during the interval $t_1 \le t \le t_3$ for an observer located in S. The simplest function $\frac{d\varphi}{dt}$ which gives time reversal is

$$\frac{d\varphi_{II}}{dt} = \frac{1}{(t_2 - t)^{\frac{1}{3}}}\sqrt{a_2 (t_2 - t)^{\frac{2}{3}} + b_2}\,. \tag{4.8}$$

Introducing Eq. (4.8) into Eq. (2.3) we have

$$\frac{\partial t'_{II}}{\partial t} = \frac{1}{(t_2 - t)^{\frac{1}{3}}} \sqrt{(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2}. \tag{4.9}$$

It is observed that $\frac{\partial t'_{II}}{\partial t}$ changes sign at $t = t_2$ as it was discussed in Eq. (2.3).
Integrating Eq. (4.9) yields

$$t'_{II} = t'_1 - \frac{1}{1 + a_2 r^2}\left[(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2\right]^{\frac{3}{2}} + \frac{1}{1 + a_2 r^2}\left[(1 + a_2 r^2)(t_2 - t_1)^{\frac{2}{3}} + b_2 r^2\right]^{\frac{3}{2}} \tag{4.10}$$

where $t'_1 = t'_1(r)$ is a function that can be calculated from the continuity of t'

$$t'_1 = t'_0 + \frac{t_1 - t_0}{2}\sqrt{1 + a_1^{\,2} r^2 (t_1 - t_0)^2} + \frac{1}{2a_1 r} Ln\left[a_1 r(t_1 - t_0) + \sqrt{1 + a_1^{\,2} r^2 (t_1 - t_0)^2}\right]. \tag{4.11}$$

In order to avoid divergences it is convenient that

$$a_2 > 0. \tag{4.12}$$

The form of the function (4.10) that reaches a maximum at $t = t_2$ allows time reversal which occurs at that time.

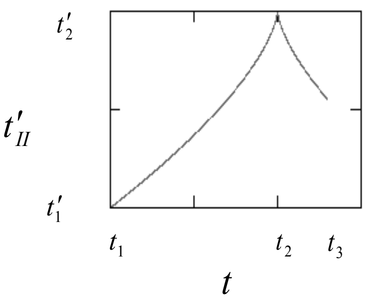

FIGURE 3. t'_{II} as function of t with fixed r during the second leg of time travel.

Deriving Eq. (4.10) and taking into account Eq. (4.11) we obtain

$$\begin{aligned}\frac{\partial t'_{II}}{\partial r} = {} & \frac{t_1 - t_0}{2r}\sqrt{1 + a_1^{\,2} r^2 (t_1 - t_0)^2} - \frac{1}{2a_1 r^2} Ln\left[a_1 r(t_1 - t_0) + \sqrt{1 + a_1^{\,2} r^2 (t_1 - t_0)^2}\right] \\ & - \frac{a_2 r(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + a_2 b_2 r^3 + 3b_2 r}{(1 + a_2 r^2)^2}\sqrt{(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2} \\ & + \frac{a_2 r(1 + a_2 r^2)(t_2 - t_1)^{\frac{2}{3}} + a_2 b_2 r^3 + 3b_2 r}{(1 + a_2 r^2)^2}\sqrt{(1 + a_2 r^2)(t_2 - t_1)^{\frac{2}{3}} + b_2 r^2} > 0.\end{aligned} \tag{4.13}$$

The proper energy density at time $t = t_1$ is obtained from Eq. (3.5)

$$\rho_1 = \rho(t_1) = \frac{\rho_0}{1 + r^2\left(\frac{d\varphi_I}{dt}\right)^2_{t=t_1}}. \tag{4.14}$$

Introducing Eqs. (4.8) and (4.14) into Eqs. (3.7) and (3.8) we obtain for this fluid in the reference frame S during the second leg of the time travel

$$f_{II}^{r} = -\rho_0 r \frac{a_2 (t_2 - t)^{\frac{2}{3}} + b_2}{(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2}, \tag{4.15}$$

$$f_{II}^{\theta} = \rho_0 \frac{r\sqrt{(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2}\,\frac{\partial t'_{II}}{\partial r}\left[a_2 (t_2 - t)^{\frac{2}{3}} + b_2\right]^{\frac{3}{2}} - \frac{b_2}{3\sqrt{a_2 (t_2 - t)^{\frac{2}{3}} + b_2}}}{\left[(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2\right]^2} \tag{4.16}$$

where $\frac{\partial t'_{II}}{\partial r}$ is given by Eq. (4.13). It is observed that f_{II}^{r} and f_{II}^{θ} are finite through the interval although the metric in Eq. (2.2) is singular at $t = t_2$.

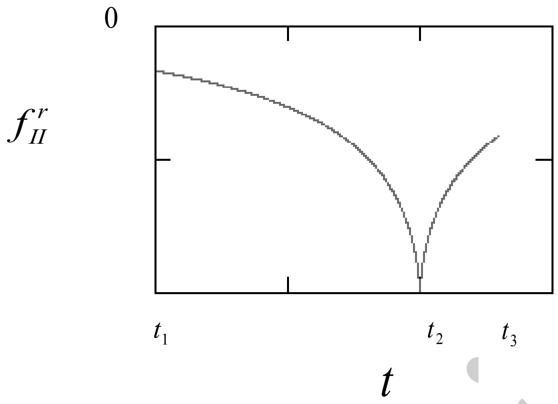

FIGURE 4. f_{II}^{r} as function of t with fixed r during the second leg of time travel.

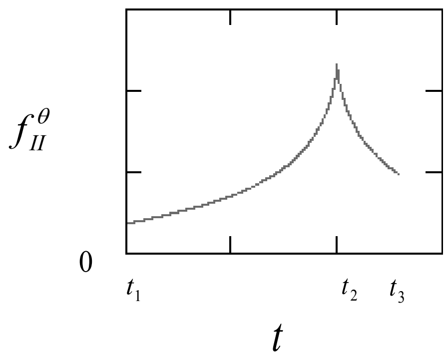

FIGURE 5. f_{II}^{θ} as function of t with fixed r during the second leg of time travel.

In order to obtain Eq. (4.16) when $t \geq t_2$, the negative sign in Eq. (2.3) has been chosen to allow the travel to the past. This choice is possible because the density of the external force f_{II}^{θ} in Eq. (3.8) or (4.16) depends on the sign of $\frac{\partial t'_{II}}{\partial t}$ since $\frac{\partial t'_{II}}{\partial r} > 0$. This means that in the reference frame S, different values of f_{II}^{θ} are obtained according to the positive or negative sign of $\frac{\partial t'_{II}}{\partial t}$. This makes the trip to the past possible, because when $t \geq t_2$, the positive and negative solutions in Eq. (2.3) are physically different in S, which allows to choose between one of them.

It can also be verified that changing the sign of the solution of Eq. (4.16) also allows the trip to the past when $t \geq t_2$. Both solutions only differ in the sign of the angular velocity $\frac{d\varphi_{II}}{dt}$.

The second leg of time travel occurs during the interval $t'_1 \leq t' \leq t'_3$ for an observer located in S´.

The rotating motion of this fluid in this reference frame during this leg is

$$f'^{r}_{II} = f^{r}_{II} = -\rho_0 r \frac{a_2 (t_2 - t)^{\frac{2}{3}} + b_2}{(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2}, \tag{4.17}$$

$$f'^{\theta}_{II} = -\frac{\rho_0 b_2}{3\sqrt{a_2 (t_2 - t)^{\frac{2}{3}} + b_2}\left[(1 + a_2 r^2)(t_2 - t)^{\frac{2}{3}} + b_2 r^2\right]^2}. \tag{4.18}$$

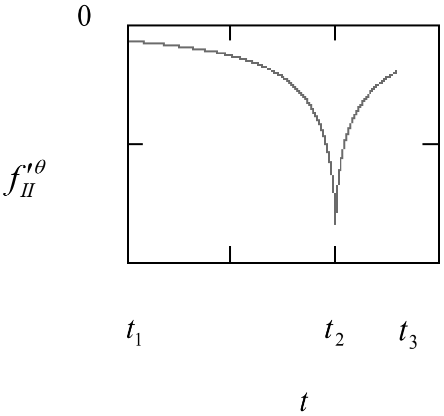

FIGURE 6. f'^{θ}_{II} as function of t with fixed r during the second leg of time travel.

We also note that f'^{r}_{II} and f'^{θ}_{II} are finite through the interval.
It is observed that the density of the external force f'^{θ}_{II} given by Eq. (3.12) or (4.18) does not depend on $\frac{\partial t'_{II}}{\partial t}$. This means that an observer located in S´ can not force the time backwards because the positive and negative solutions of Eq. (2.3) lead to the same f'^{θ}_{II}. For this reason, f'^{θ}_{II} only provides $\frac{\partial t'_{II}}{\partial t} > 0$. On the other hand, an observer located in the reference frame S, physically distinguishes both solutions and he can choose the solution in Eq. (3.8) or (4.16) that causes the time reversal.
It can be advisable to write Eqs. (4.17) and (4.18) as a function of time t' measured in S´. For this, it is necessary to find t as a function of t' from Eq. (4.10).
It can be checked by inspection of Eqs. (3.14), (4.8) and (3.5) that the fluid reaches the speed of light in S´ at time $t = t_2$ because the proper energy density of the fluid can be null at that time in which time reversal occurs.

Back to the Past

The third leg of time travel occurs during the interval $t_3 \le t \le t_4$ for an observer located in S. It is convenient to keep angular velocity $\frac{d\varphi}{dt}$ constant during this leg and thus

$$\frac{d\varphi_{III}}{dt} = a_3, \tag{4.19}$$

where a_3 is a constant so that

$$a_3 < 0. \tag{4.20}$$

In addition

$$\frac{\partial t'_{III}}{\partial t} = \sqrt{1 + a_3^{\ 2} r^2} \tag{4.21}$$

and

$$t'_{III} = t'_3 + \sqrt{1 + a_3^{\ 2} r^2}\left(t - t_3\right) \tag{4.22}$$

where

$$t'_3 = t'_0 + \frac{t_1 - t_0}{2}\sqrt{1 + a_1^{\ 2} r^2 \left(t_1 - t_0\right)^2} + \frac{1}{2a_1 r} Ln\left[a_1 r\left(t_1 - t_0\right) + \sqrt{1 + a_1^{\ 2} r^2 \left(t_1 - t_0\right)^2}\right]$$

$$-\frac{1}{1 + a_2 r^2}\left[\left(1 + a_2 r^2\right)\left(t_2 - t_3\right)^{\frac{2}{3}} + b_2 r^2\right]^{\frac{3}{2}} + \frac{1}{1 + a_2 r^2}\left[\left(1 + a_2 r^2\right)\left(t_2 - t_1\right)^{\frac{2}{3}} + b_2 r^2\right]^{\frac{3}{2}}. \tag{4.23}$$

Deriving Eq. (4.22) we obtain

$$\frac{\partial t'_{III}}{\partial r} = \frac{dt'_3}{dr} + \frac{a_3^{\ 2} r}{\sqrt{1 + a_3^{\ 2} r^2}}\left(t - t_3\right). \tag{4.24}$$

It is necessary to determine the velocity (4.21) of the travel backward in time. As $\frac{\partial t'_{III}}{\partial t}$ depends on r to determine this velocity it is convenient to fix r so that, in $r = r_0$ verifies

$$\left.\frac{\partial t'_{III}}{\partial t}\right|_{r_0, t_3} = -V \tag{4.25}$$

where V is a constant greater than zero

$$V > 0. \tag{4.26}$$

We obtain for this fluid in the reference frame S during the third leg of time travel

$$f_{III}^{r} = -\rho_0 r \frac{a_3^{\ 2}}{1 + a_3^{\ 2} r^2}, \tag{4.27}$$

$$f_{III}^{\theta} = \rho_0 \frac{r\sqrt{1 + a_3^{\ 2} r^2}\,\frac{\partial t'_{III}}{\partial r}\, a_3^{\ 3}}{\left[1 + +a_3^{\ 2} r^2\right]^2} \tag{4.28}$$

where $\frac{\partial t'_{III}}{\partial r}$ is given by Eqs. (4.23) and (4.24).

New Time Reversal

The fourth leg of time travel occurs during the interval $t_4 \le t \le t_6$ for an observer located in S. The simplest function $\frac{d\varphi}{dt}$ which gives a new time reversal is

$$\frac{d\varphi_{IV}}{dt}=\frac{1}{(t-t_5)^{\frac{1}{3}}}\sqrt{a_4(t-t_5)^{\frac{2}{3}}+b_4}\,. \tag{4.29}$$

where t_5, a_4 and b_4 are constants verifying

$$t_4<t_5<t_6\,;\;\; b_4>0\,;\;\; b_4>-a_4(t_4-t_5)^{\frac{2}{3}}. \tag{4.30}$$

Introducing Eq. (4.29) into Eq. (2.3) we have

$$\frac{\partial t'_{IV}}{\partial t}=\frac{1}{(t-t_5)^{\frac{1}{3}}}\sqrt{(1+a_4r^2)(t-t_5)^{\frac{2}{3}}+b_4r^2}\,. \tag{4.31}$$

It is observed that $\frac{\partial t'_{IV}}{\partial t}$ changes sign at $t=t_5$ as it was discussed in Eq. (2.3).

Integrating Eq. (4.31) yields

$$t'_{IV}=t'_4+\frac{1}{1+a_4r^2}\left[(1+a_4r^2)(t-t_5)^{\frac{2}{3}}+b_4r^2\right]^{\frac{3}{2}}-\frac{1}{1+a_4r^2}\left[(1+a_4r^2)(t_4-t_5)^{\frac{2}{3}}+b_4r^2\right]^{\frac{3}{2}} \tag{4.32}$$

where

$$\begin{aligned} t'_4 &= t'_0+\frac{t_1-t_0}{2}\sqrt{1+a_1^2r^2(t_1-t_0)^2}+\frac{1}{2a_1r}Ln\left[a_1r(t_1-t_0)+\sqrt{1+a_1^2r^2(t_1-t_0)^2}\right] \\ &-\frac{1}{1+a_2r^2}\left[(1+a_2r^2)(t_2-t_3)^{\frac{2}{3}}+b_2r^2\right]^{\frac{3}{2}}+\frac{1}{1+a_2r^2}\left[(1+a_2r^2)(t_2-t_1)^{\frac{2}{3}}+b_2r^2\right]^{\frac{3}{2}} \\ &+\frac{1}{(t_2-t_3)^{\frac{1}{3}}}\sqrt{(1+a_2r^2)(t_2-t_3)^{\frac{2}{3}}+b_2r^2}\,(t_4-t_3). \end{aligned} \tag{4.33}$$

In order to avoid divergences in Eq. (4.32) it is convenient that

$$a_4>0. \tag{4.34}$$

The form of the function (4.32) that reaches a minimum at $t=t_5$ allows a new time reversal which occurs at that time.

Deriving Eq. (4.32) we obtain

$$\begin{aligned} \frac{\partial t'_{IV}}{\partial r} &= \frac{dt'_4}{dr}-\frac{a_4r(1+a_4r^2)(t_4-t_5)^{\frac{2}{3}}+a_4b_4r^3+3b_4r}{(1+a_4r^2)^2}\sqrt{(1+a_4r^2)(t_4-t_5)^{\frac{2}{3}}+b_4r^2} \\ &+\frac{a_4r(1+a_4r^2)(t-t_5)^{\frac{2}{3}}+a_4b_4r^3+3b_4r}{(1+a_4r^2)^2}\sqrt{(1+a_4r^2)(t-t_5)^{\frac{2}{3}}+b_4r^2}<0. \end{aligned} \tag{4.35}$$

The proper energy density at time $t=t_4$ is calculated from Eq. (3.5)

$$\rho_4=\rho(t_4)=\frac{\rho_0}{1+r^2\left(\frac{d\varphi_{III}}{dt}\right)^2_{t=t_4}}. \tag{4.36}$$

Introducing Eqs. (4.29) and (4.36) into Eqs. (3.7) and (3.8) we obtain for this fluid in the reference frame S during the fourth leg of time travel

$$f^r_{IV}=-\rho_0r\frac{a_4(t-t_5)^{\frac{2}{3}}+b_4}{(1+a_4r^2)(t-t_5)^{\frac{2}{3}}+b_4r^2}, \tag{4.37}$$

$$f_{IV}^{\theta}=\rho_0\,\frac{r\sqrt{\left(1+a_4r^2\right)\left(t-t_5\right)^{\frac{2}{3}}+b_4r^2}\,\dfrac{\partial t'_{IV}}{\partial r}\left[a_4\left(t-t_5\right)^{\frac{2}{3}}+b_4\right]^{\frac{3}{2}}+\dfrac{b_4}{3\sqrt{a_4\left(t-t_5\right)^{\frac{2}{3}}+b_4}}}{\left[\left(1+a_4r^2\right)\left(t-t_5\right)^{\frac{2}{3}}+b_4r^2\right]^2} \tag{4.38}$$

where $\frac{\partial t'_{IV}}{\partial r}$ is given by Eqs. (4.33) and (4.35). We also note that f_{IV}^{r} and f_{IV}^{θ} are finite through the interval although the metric in Eq. (2.2) is singular at $t=t_5$.

Again, it is observed that the new time reversal is possible because f_{IV}^{θ} depends on the sign of $\frac{\partial t'_{IV}}{\partial t}$ because $\frac{\partial t'_{IV}}{\partial r}<0$.

We can prove that $f_{IV}'^{r}$ and $f_{IV}'^{\theta}$ are finite through the interval. Again, it can be verified that at time $t=t_5$ in S, the fluid reaches the speed of light in S′.

Temporal Deceleration

The fifth leg of time travel occurs during the interval $t_6\le t\le T$ for an observer located in S. If the rotation stops at the end of the travel, at time $t=T$ in S, the simplest function $\frac{d\varphi}{dt}$ is

$$\frac{d\varphi_V}{dt}=a_5\left(t-T\right) \tag{4.39}$$

where a_5 is a constant such that

$$a_5<0. \tag{4.40}$$

In addition

$$\frac{\partial t'_V}{\partial t}=\sqrt{1+a_5^{\,2}r^2\left(t-T\right)^2} \tag{4.41}$$

and

$$\begin{aligned} t'_V = t'_6 &-\frac{t_6-T}{2}\sqrt{1+a_5^{\,2}r^2\left(t_6-T\right)^2}-\frac{1}{2a_5r}Ln\left[a_5r\left(t_6-T\right)+\sqrt{1+a_5^{\,2}r^2\left(t_6-T\right)^2}\right]\\ &+\frac{t-T}{2}\sqrt{1+a_5^{\,2}r^2\left(t-T\right)^2}+\frac{1}{2a_5r}Ln\left[a_5r\left(t-T\right)+\sqrt{1+a_5^{\,2}r^2\left(t-T\right)^2}\right]. \end{aligned} \tag{4.42}$$

where

$$\begin{aligned} t'_6=t'_0&+\frac{t_1-t_0}{2}\sqrt{1+a_1^{\,2}r^2\left(t_1-t_0\right)^2}+\frac{1}{2a_1r}Ln\left[a_1r\left(t_1-t_0\right)+\sqrt{1+a_1^{\,2}r^2\left(t_1-t_0\right)^2}\right]\\ &-\frac{1}{1+a_2r^2}\left[\left(1+a_2r^2\right)\left(t_2-t_3\right)^{\frac{2}{3}}+b_2r^2\right]^{\frac{3}{2}}+\frac{1}{1+a_2r^2}\left[\left(1+a_2r^2\right)\left(t_2-t_1\right)^{\frac{2}{3}}+b_2r^2\right]^{\frac{3}{2}}\\ &+\frac{1}{\left(t_2-t_3\right)^{\frac{1}{3}}}\sqrt{\left(1+a_2r^2\right)\left(t_2-t_3\right)^{\frac{2}{3}}+b_2r^2}\left(t_4-t_3\right)+\frac{1}{1+a_4r^2}\left[\left(1+a_4r^2\right)\left(t_6-t_5\right)^{\frac{2}{3}}+b_4r^2\right]^{\frac{3}{2}} \end{aligned}$$

$$-\frac{1}{1+a_4r^2}\left[\left(1+a_4r^2\right)\left(t_4-t_5\right)^{\frac{2}{3}}+b_4r^2\right]^{\frac{3}{2}}. \tag{4.43}$$

It is convenient that the travel ends at a certain time T' in the past of S´. As $t'=t'(t,r)$, it is advisable to fix r in which this happens. This will be $r=r_0$ and so

$$t'_V(T)_{r_0}=T' \tag{4.44}$$

Deriving Eq. (4.42) we obtain

$$\frac{\partial t'_V}{\partial r}=\frac{dt'_6}{dr}-\frac{t_6-T}{2r}\sqrt{1+a_5{}^2r^2\left(t_6-T\right)^2}+\frac{1}{2a_5r^2}Ln\left[a_5r\left(t_6-T\right)+\sqrt{1+a_5{}^2r^2\left(t_6-T\right)^2}\right]$$
$$+\frac{t-T}{2r}\sqrt{1+a_5{}^2r^2\left(t-T\right)^2}-\frac{1}{2a_5r^2}Ln\left[a_5r\left(t-T\right)+\sqrt{1+a_5{}^2r^2\left(t-T\right)^2}\right]. \tag{4.45}$$

We obtain for this fluid in the reference frame S during the fifth leg of time travel

$$f_V^r=-\frac{\rho_0a_5{}^2r\left(t-T\right)^2}{1+a_5{}^2r^2\left(t-T\right)^2}, \tag{4.46}$$

$$f_V^\theta=\rho_0\frac{r\sqrt{1+a_5{}^2r^2\left(t-T\right)^2}\,\dfrac{\partial t'_V}{\partial r}\,a_5{}^3\left(t-T\right)^3-a_5}{\left[1+a_5{}^2r^2\left(t-T\right)^2\right]^2} \tag{4.47}$$

where $\frac{\partial t'_V}{\partial r}$ is given by Eqs. (4.43) and (4.45). Observe that the constants appearing in the equations have been chosen to avoid divergences and they will be constrained by Eqs. (4.25), (4.44) and continuity conditions.

And this is the end of the travel to the past. As it is easy to imagine, the return travel to the present can be carried out in a similar way in only three legs that do not entail as much difficulty as the travel to the past.

Finally, note that the travel to the past can be used in interstellar space travel. Indeed, due to the effect of time dilation, if a traveller reached a speed close to that of light he could reach his destination star in a few months. However, when he returned to Earth he will note that here hundreds or thousands of years have passed. If the traveler went back in time he could make the duration of the trip the same for both the traveller and people on Earth.

APPENDIX: EINSTEIN'S EQUATIONS WITH SCALAR FIELD

The above results are adequate in a flat space-time in the absence of gravity. In practice, this happens when the gravitational field created by the fluid is so weak that it barely curves space-time. Although it is not relevant in the exposed developments, it is convenient to generalize the formalism to the case in which the gravitational field generated by the fluid is important. The results in the absence of gravity are obtained as a particular case. To do this, it is necessary to resort to Einstein's field equations.

First, it should be noted that the energy-momentum tensor (3.1) is not conserved, as it is deduced from Eq. (1.2), so that it is not compatible with Einstein's equations

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=-8\pi GT^{\mu\nu} \tag{A.1}$$

for which the energy-momentum tensor is conserved. This means that the energy-momentum tensor (3.1) is not adequate to calculate the gravitational field created by a perfect fluid subjected to mechanical forces.

The simplest way to avoid this difficulty is to introduce a scalar field ϕ into Einstein's Eqs. (A.1). This scalar field verifies a new differential equation. The simplest generally covariant field equation for such a scalar field is

$$\phi_{;\ ;\alpha}^{\ \alpha} = 4\pi\lambda T_{M\alpha}^{\alpha} \tag{A.2}$$

where λ is a coupling constant and $T_M^{\mu\nu}$ is the energy-momentum tensor of the matter. The most general symmetric tensor for this field must contain terms which involves two derivatives of one or two ϕ fields

$$T_{\phi}^{\mu\nu} = A(x)\phi_{;}^{\ \mu}\phi_{;}^{\ \nu} + B(x)g^{\mu\nu}\phi_{;}^{\ \alpha}\phi_{;\alpha} + C(x)\phi_{;\ ;}^{\ \mu\ \nu} + D(x)g^{\mu\nu}\phi_{;\ ;\alpha}^{\ \alpha}. \tag{A.3}$$

Einstein's Eqs. (A.1) including this scalar field are

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -8\pi G\left(T_M^{\ \mu\nu} + T_{\phi}^{\mu\nu}\right) \tag{A.4}$$

where $T_M^{\mu\nu}$ is the energy-momentum tensor of the matter verifying

$$f^{\mu} = T_{M\ ;\alpha}^{\mu\alpha} \tag{A.5}$$

and $T_{\phi}^{\mu\nu}$ is the energy-momentum tensor of the scalar field given by Eqs. (A.3).

In order to determine the functions $A(x)$, $B(x)$, $C(x)$ and $D(x)$ which appear in Eqs. (A.3), the covariant divergence of Eqs. (A.4) is calculated obtaining

$$f^{\mu} = -\frac{\partial A}{\partial x^{\alpha}}\phi_{;}^{\mu}\phi_{;}^{\alpha} - \left[A(x) + 2B(x)\right]\phi_{;\ ;}^{\ \mu\ \alpha}\phi_{;\alpha} - A(x)\phi_{;}^{\mu}\phi_{;\ ;\alpha}^{\alpha} - \frac{\partial B}{\partial x^{\beta}}g^{\mu\beta}\phi_{;}^{\alpha}\phi_{;\alpha}$$

$$-\frac{\partial C}{\partial x^{\alpha}}\phi_{;\ ;}^{\ \mu\ \alpha} - C(x)\phi_{;\alpha;\ ;}^{\ \ \ \alpha\ \mu} - \frac{\partial D}{\partial x^{\beta}}g^{\mu\beta}\phi_{;\ ;\alpha}^{\alpha} - D(x)\phi_{;\ ;\ ;\alpha}^{\ \mu\ \alpha}\ . \tag{A.6}$$

where Eqs. (A.3) and (A.5) have been used.

These are the equations of the dynamics of the scalar field that determine the four functions $A(x)$, $B(x)$, $C(x)$ and $D(x)$.

Reference Frames

The ten Eqs. (A.4) of the gravitational field are not independent but are related by the four Bianchi identities which reduces Eqs. (A.4) to six independent equations. To determine an unambiguous metric, it is necessary to add four more equations to fix the Gauge, that is, the reference frame.

In the presence of gravity, it is also convenient to define a reference frame at rest S´ as described in section I. Since the Principle of General Covariance tells that Eqs. (1.2) are also applicable in the case of gravity, then in any reference frame, for example, the reference frame S´, the equations of motion of the fluid will be

$$f'^{\mu} = T'^{\mu\alpha}_{M\ ;\alpha} = \frac{\partial T'^{\mu\alpha}_{M}}{\partial x'^{\alpha}} + \Gamma'^{\alpha}_{\alpha\beta}T'^{\mu\beta}_{M} + \Gamma'^{\mu}_{\alpha\beta}T'^{\alpha\beta}_{M}. \tag{A.7}$$

Now, in the reference frame at rest S´, the Eqs. (1.1) must be also verified

$$f'^{\mu} = \frac{\partial T'^{\mu\alpha}_{M}}{\partial x'^{\alpha}}. \tag{A.8}$$

This equates, in view of Eqs. (A.7) to choose a gauge such that

$$\Gamma'^{\alpha}_{\alpha\beta}T'^{\mu\beta}_{M} + \Gamma'^{\mu}_{\alpha\beta}T'^{\alpha\beta}_{M} = 0. \tag{A.9}$$

These four equations that determine the Gauge, that is, the reference frame S´, are not covariant so they are not valid in any reference frame, but only in S´.

The moving reference frame S is chosen, as in section I, such that the fluid, which originates the gravitational field, is at rest[6]

$$\frac{dx^i}{d\tau} = 0. \tag{A.10}$$

In addition, to fix completely the reference frame S, it is necessary to add another condition that it can be obtained as in Eq. (1.8) by imposing that time-time component of the metric must be the same in any reference frame

$$g_{tt} = g'_{tt} \tag{A.11}$$

where g'_{tt} is the time-time component of the metric in the reference frame S´ at rest. The three Eqs. (A.10) together with Eq. (A.11) fix another Gauge, that is, the reference frame S.

Relevance of the Scalar Field

The introduction of the scalar field ϕ gives the Minkowski metric $\eta_{\mu\nu}$ as a solution of Einstein's Eqs. (A.4) in the Gauge Eqs. (A.9). This solution is adequate in the absence of gravity. In practice, this happens when the gravitational field created by the fluid is so weak that it barely curves space-time. In this situation, in any reference frame, it is verified

$$R^{\mu\nu} = 0 \tag{A.12}$$

and

$$R = g_{\alpha\beta} R^{\alpha\beta} = 0. \tag{A.13}$$

It can be shown for this fluid, with Eqs. (A.12), (A.13) and (A.4)

$$T_M^{\mu\nu} = -T_\phi^{\mu\nu}, \tag{A.14}$$

so the ϕ field is especially important.
In vacuum, the energy-momentum tensor of the matter is zero

$$T_M^{\mu\nu} = 0 \tag{A.15}$$

and Eq. (A.2) will be written as

$$\phi_{;\ ;\alpha}^{\ \alpha} = 0 \tag{A.16}$$

while Eqs. (A.5) give

$$f^{\mu} = 0. \tag{A.17}$$

Introducing Eqs. (A.16) and (A.17) into Eqs. (A.6) we obtain

$$0 = \frac{\partial A}{\partial x^{\alpha}} \phi_;^{\mu} \phi_;^{\alpha} + \left[A(x) + 2B(x)\right] \phi_{;\ ;}^{\mu\ \alpha} \phi_{;\alpha} + \frac{\partial B}{\partial x^{\beta}} g^{\mu\beta} \phi_;^{\alpha} \phi_{;\alpha} + \frac{\partial C}{\partial x^{\alpha}} \phi_{;\ ;}^{\mu\ \alpha} + C(x) \phi_{;\alpha;\ ;}^{\ \ \ \alpha\ \mu}. \tag{A.18}$$

The solutions of these equations are

$$A(x) = B(x) = C(x) = 0. \tag{A.19}$$

With Eqs. (A.16) and (A.19) the energy-momentum tensor (A.3) of the scalar field ϕ in vacuum is

$$T_\phi^{\mu\nu} = 0. \tag{A.20}$$

Equations (A.15) and (A.20) imply that Eqs. (A.1) and (A.4) have the same solutions in vacuum, that is, in this case the scalar field does not alter the solutions of the Eqs. (A.1) that explain, among other phenomena, the dynamics of the planets. In other cases, their significance will have to be determined.

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[6] It is possible to find a reference frame like this because external forces can apply in such a way that all the particles of the fluid are at rest.

CONCLUSIONS

The exposed developments demonstrate the theoretical possibility of travelling to the past, in the case of a macroscopic perfect fluid without pressure in the absence of gravity and without violating the laws of Special Relativity. This is only possible if the fluid is accelerated to the speed of light. To demonstrate this, a relativistic treatment of rotation has been made using the principle of General Covariance, which has proved to be practical. Finally, it is necessary to introduce a Scalar Field in Einstein's equations to explain relativistic dynamics satisfactorily. These equations have made possible to generalize the formalism to the case in which the fluid generates an appreciable Gravitational Field.

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